Solution Challenge Problem 2

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SOLUTION TRAPEZOID

1. SOLUTION: METHOD BY TRIGONOMETRY

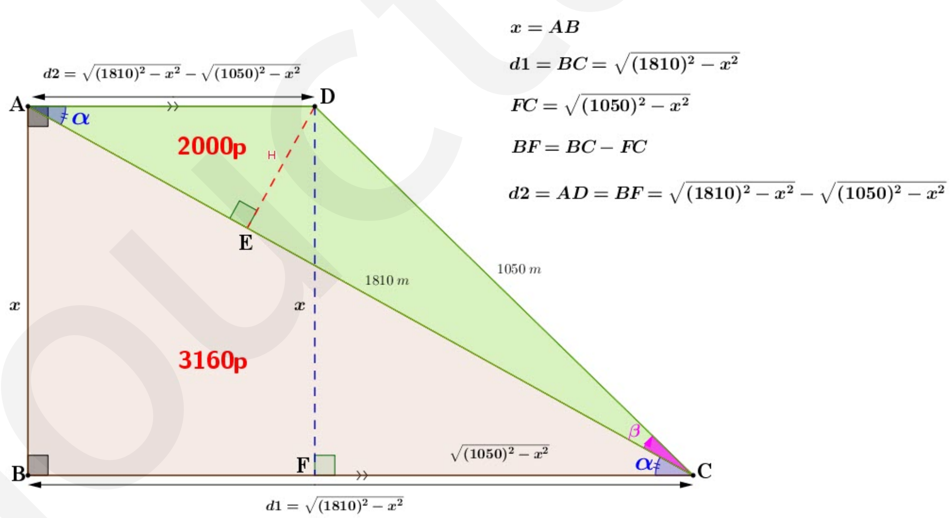

Figure 1: Trapezoid.

Let $AB = x$

We also have $BC = d_1$

From the right triangle $\triangle ABC$ we have the hypothenuse $AC = 1810$.

We have:

$d_1 = \sqrt{(1810)^2 - x^2}$

We also have another right triangle $\triangle DFC$

$FC = \sqrt{(1050)^2 - x^2}$

Another equality:

$AD = d_2 = BF = BC - FC$

We can also write

$d_2 = \sqrt{(1810)^2 - x^2} - \sqrt{(1050)^2 - x^2}$

We can also see that :

$\angle BCA \cong \angle CAD$

$m\angle BCA = \alpha = m\angle CAD$

The Area $\mathcal{A}$ $\triangle ABC$ will contain 3160 plots of land:

$\mathcal{A}\ \triangle ABC = 3160p$

Furthermore

$\mathcal{A}\ \triangle ACD = 2000p$

We use the formula of two sides and the included angle:

For triangle $\triangle ABC$:

$$\frac{1}{2} \cdot BC \cdot AC \cdot \sin\alpha = 3160p \tag{1}$$

For $\triangle ACD$:

$$\frac{1}{2} \cdot AD \cdot AC \cdot \sin\alpha = 2000p \tag{2}$$

Let's divide (1) by (2)

$$\frac{\frac{1}{2}\cdot BC\cdot AC\cdot \sin\alpha}{\frac{1}{2}\cdot AD\cdot AC\cdot \sin\alpha}=\frac{3160p}{2000p}$$

Simplifying we get:

$\frac{BC}{AD}=\frac{3.16}{2}$

Or

$\frac{BC}{AD}=1.58$

But $d_1 = BC$ and $d_2 = AD$

This yields:

$$\frac{\sqrt{(1810)^2-x^2}}{\sqrt{(1810)^2-x^2}-\sqrt{(1050)^2-x^2}}=1.58$$

Multiplying the two sides by the denominator of the first side:

$\sqrt{(1810)^2-x^2}=1.58(\sqrt{(1810)^2-x^2}-\sqrt{(1050)^2-x^2})$

$\sqrt{(1810)^2-x^2}=1.58(\sqrt{(1810)^2-x^2})-1.58(\sqrt{(1050)^2-x^2})$

$(1.58-1)\sqrt{(1810)^2-x^2}=1.58\sqrt{(1050)^2-x^2}$

$0.58\sqrt{(1810)^2-x^2}=1.58\sqrt{(1050)^2-x^2}$

$\sqrt{(1810)^2-x^2}=\frac{1.58}{0.58}\sqrt{(1050)^2-x^2}$

$\sqrt{(1810)^2-x^2}=\frac{0.79}{0.29}\sqrt{(1050)^2-x^2}$

Let $k=\frac{0.79}{0.29}$

We get:

$\sqrt{(1810)^2-x^2}=k\sqrt{(1050)^2-x^2}$

Squaring both sides we get:

$(1810)^2-x^2=k^2((1050)^2-x^2)$

$(1810)^2-x^2=k^2(1050)^2-k^2x^2$

$(k^2-1)x^2=k^2\cdot(1050)^2-(1810)^2$

$x^2=\frac{k^2\cdot(1050)^2-(1810)^2}{k^2-1}$

$x=\sqrt{\frac{k^2\cdot(1050)^2-(1810)^2}{k^2-1}}$

We plugin the values:

$$x = \sqrt{\frac{(\frac{0.79}{0.29})^2 \cdot (1050)^2 - (1810)^2}{(\frac{0.79}{0.29})^2 - 1}}$$

$x = 874.0605963$

Rounding:

$\boxed{x = 874\ m}$

Finding the angles α and β

$\sin\alpha = \frac{x}{1810}$

We get

$\sin\alpha = \frac{874.0605963}{1810} \Leftarrow \alpha = 28^\circ.8753967$

On the other hand, in trianglee $\triangle ADC$:

$$\frac{\sin(180^\circ - (\alpha + \beta))}{1810} = \frac{\sin\alpha}{1050} \Rightarrow \frac{\sin(\alpha + \beta)}{1810} = \frac{\sin\alpha}{1050}$$

This yields:

$\sin(\alpha + \beta) = \frac{1810}{1050} \sin\alpha$

Or:

$\sin(\alpha + \beta) = \frac{1810}{1050} \sin 28^\circ.8753967 \Leftarrow (\alpha + \beta) = 56^\circ.35006873$

So we get:

$\beta = 56^\circ.35006873 - 28^\circ.8753967$

$\beta = 27^\circ.47467203$

The area of the triangles:

Area $\mathcal{A}\ \triangle ABC = \frac{1}{2} \cdot x \cdot AC \cdot \cos\alpha$

Area $\mathcal{A}\ \triangle ABC = \dfrac{1}{2} \times 874.0605963 \times 1810 \times \cos 28^\circ.8753967$

Area $\mathcal{A}\ \triangle ABC = 692678.2811$

Rounding:

$\boxed{\text{Area } \mathcal{A} \ \triangle ABC = 692678 \ m^2}$

For $\triangle ADC$

Area $\mathcal{A}\triangle ADC = \frac{1}{2} \cdot 1050 \cdot 1810 \cdot \sin\beta$

Or:

Area $\mathcal{A}\triangle ADC = \dfrac{1}{2} \cdot 1050 \cdot 1810 \cdot \sin 27^\circ.47467203$

Area $\mathcal{A}\triangle ADC = 438403.9754$

Rounding:

$\boxed{\text{Area } \mathcal{A}\triangle ADC = 438404 \ m^2}$

AIRE SURFACE TOTALE :

Adding the two areas:

$\mathcal{A} \ \square ABCD = 692678 + 438404$

$\boxed{\mathcal{A} \ \square ABCD = 1131082 \ m^2}$

$\mathcal{A}$ *AREA OF PLOT* : 5160 *plots in total*

Area Plot $= \frac{1131082}{5160}$

$\boxed{\mathcal{A} \ \textit{AREA OF PLOT} := 219 \ m^2}$

Heron formula for the triangle $\triangle ADC$

Calculating d_2

$$\frac{\sin\alpha}{1050} = \frac{\sin\beta}{d_2} \Rightarrow d_2 = 1050 \cdot \frac{\sin\beta}{\sin\alpha}$$

$$d_2 = 1050 \cdot \frac{\sin 27^\circ.47467203}{\sin 28^\circ.8753967}$$

$d_2 = 1003.143208$

The sides : 1050, 1810, 1003.143208

Calculating s:

$s = \frac{1050+1810+1003.143208}{2}$

$s = 1931.571604$

$s - 1050 = 1931.571604 - 1050 = 881.571604$

$s - 1810 = 1931.571604 - 1810 = 121.571604$

$s - 1003.143208 = 1931.571604 - 1003.143208 = 928.428396$

Area $\mathcal{A} \triangle ADC = \sqrt{1931.571604 \times 881.571604 \times 121.571604 \times 928.428396}$

Area $\mathcal{A} \triangle ADC = 438403.9751$

Rounding:

$\boxed{\text{L'aire } \mathcal{A}\triangle ADC = 438404\ m^2}$

2. SOLUTION: ALGEBRA METHOD

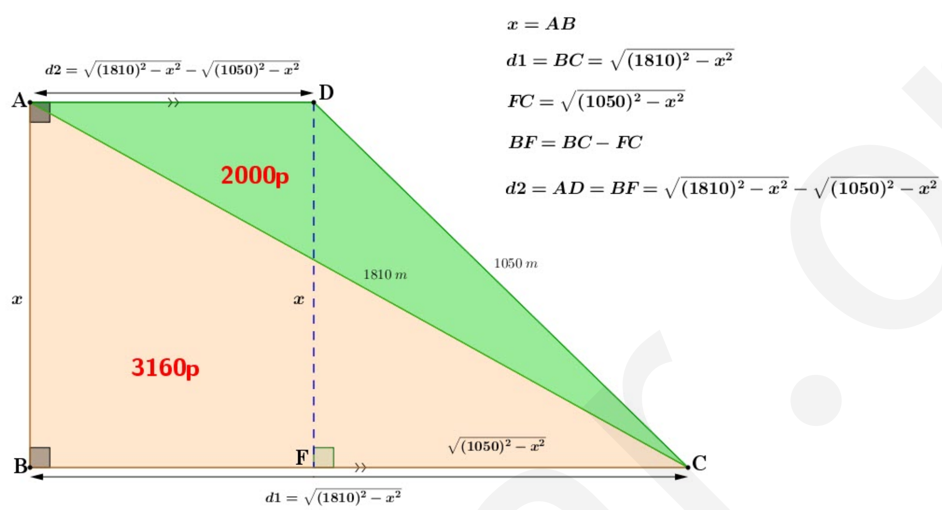

Figure 2: Trapezoid2.

Let $AB = x$

Also $BC = d_1$

For right triangle $\triangle ABC$ we have hypothenuse $AC = 1810$.

We have:

$d_1 = \sqrt{(1810)^2 - x^2}$

We have another right triangle $\triangle DFC$

$FC = \sqrt{(1050)^2 - x^2}$

Another equality:

$AD = d_2 = BF = BC - FC$

We can also write:

$d_2 = \sqrt{(1810)^2 - x^2} - \sqrt{(1050)^2 - x^2}$

Area of the surface $\mathcal{A}\triangle ABC$will contain 3160 plots out of 5160.

For $\triangle ABC$:

$\mathcal{A}\triangle ABC$

The area of $\mathcal{A}\triangle ABC = \frac{1}{2} \cdot x \cdot BC$

OR

$\mathcal{A}\triangle ABC = \frac{1}{2} \cdot x \cdot d_1$

$\mathcal{A}\triangle ABC = \frac{1}{2}x\sqrt{(1810)^2 - x^2}$

Now we get the total area:

$\mathcal{A}\ \square ABCD = \square ABFD + \triangle DFC$

Area of $\mathcal{A}\ \square ABFD$:

Area of $\mathcal{A}\ \square ABFD = x \cdot d_2$

We replace

$\mathcal{A}\ \square ABFD = x \cdot (\sqrt{(1810)^2 - x^2} - \sqrt{(1050)^2 - x^2})$

$\mathcal{A}\ \square ABFD = x\sqrt{(1810)^2 - x^2} - x\sqrt{(1050)^2 - x^2}$

For triangle $\triangle DFC$:

Area $\mathcal{A}\ \triangle DFC = \frac{1}{2} \cdot x \cdot FC$

Or

Area of $\mathcal{A}\ \triangle DFC = \frac{1}{2} \cdot x \cdot d_1$

Area of $\mathcal{A}\ \triangle DFC = \frac{1}{2} \cdot x \cdot \sqrt{(1050)^2 - x^2}$

Total Area :

Area $\mathcal{A}\ \square ABCD = \square ABFD + \triangle DFC$

We plugin:

Area $\mathcal{A}\ \square ABCD = x\sqrt{(1810)^2 - x^2} - x\sqrt{(1050)^2 - x^2} + \dfrac{1}{2}x\sqrt{(1050)^2 - x^2}$

Area $\mathcal{A}\ \square ABCD = x\sqrt{(1810)^2 - x^2} - \frac{1}{2}x\sqrt{(1050)^2 - x^2}$

Comparing the Areas:

Area $\triangle ABC = 3160p$

Area $\square ABCD = 5160p$

It yields:

$\frac{Area\ \square ABCD}{Area\ \triangle ABC} = \frac{5160p}{3160p}$

Or:

$\mathcal{A}\ \triangle ABC = \frac{3160p}{5160p} Area\ \square ABCD$

We simplify:

$\mathcal{A}\ \triangle ABC = \frac{79}{129} Area\ \square ABCD$

We plugin:

$$\frac{1}{2}x\sqrt{(1810)^2 - x^2} = \frac{79}{129}\left(x\sqrt{(1810)^2 - x^2} - \frac{1}{2}x\sqrt{(1050)^2 - x^2}\right)$$

We can write:

$\frac{129}{158}x\sqrt{(1810)^2 - x^2} = x\sqrt{(1810)^2 - x^2} - \frac{1}{2}x\sqrt{(1050)^2 - x^2}$

Let's divide the two sides by x

$\frac{129}{158}\sqrt{(1810)^2 - x^2} = \sqrt{(1810)^2 - x^2} - \frac{1}{2}\sqrt{(1050)^2 - x^2}$

Adjusting:

$\frac{1}{2}\sqrt{(1050)^2 - x^2} = (1 - \frac{129}{158})\sqrt{(1810)^2 - x^2}$

$\frac{1}{2}\sqrt{(1050)^2 - x^2} = \frac{29}{158}\sqrt{(1810)^2 - x^2}$

Multiplying the two sides by 2:

$\sqrt{(1050)^2 - x^2} = \frac{29}{79}\sqrt{(1810)^2 - x^2}$

We can also write:

$\frac{0.79}{0.29}\sqrt{(1050)^2 - x^2} = \sqrt{(1810)^2 - x^2}$

Let $k = \frac{0.79}{0.29}$

We have:

$\sqrt{(1810)^2 - x^2} = k\sqrt{(1050)^2 - x^2}$

Squaring the two sides:

$(1810)^2 - x^2 = k^2((1050)^2 - x^2)$

$(1810)^2 - x^2 = k^2(1050)^2 - k^2x^2$

$(k^2 - 1)x^2 = k^2 \cdot (1050)^2 - (1810)^2$

$x^2 = \frac{k^2 \cdot (1050)^2 - (1810)^2}{k^2 - 1}$

$x = \sqrt{\frac{k^2 \cdot (1050)^2 - (1810)^2}{k^2 - 1}}$

We plugin:

$$x = \sqrt{\frac{(\frac{0.79}{0.29})^2 \cdot (1050)^2 - (1810)^2}{(\frac{0.79}{0.29})^2 - 1}}$$

$x = 874.0605963$

Rounding:

$\boxed{x = 874\ m}$

Areas:

The other sides:

$d_1 = \sqrt{(1810)^2 - x^2}$

$d_1 = \sqrt{(1810)^2 - (874.0605963)^2}$

$d_1 = 1584.966269$

$\boxed{d_1 = 1585\ m}$

$d_2 = \sqrt{(1810)^2 - x^2} - \sqrt{(1050)^2 - x^2}$

$d_2 = \sqrt{(1810)^2 - (874.0605963)^2} - \sqrt{(1050)^2 - (874.0605963)^2}$

$d_2 = 1003.143208$

$\boxed{d_2 = 1003\ m}$

Area $\triangle ABC = \frac{1}{2}x \cdot d_1$

Area $\triangle ABC = \frac{1}{2} \times 874.0605963 \times 1584.966269$

Area $\triangle ABC = 692678.2811$

Rounding:

$\boxed{\mathcal{A}\ \triangle ABC = 692678\ m^2}$

TOTAL AREA :

The trapezoid:

$\mathcal{A}\ \square ABCD = \frac{d_1+d_2}{2} \cdot x$

$\mathcal{A}\ \square ABCD = \frac{1584.966269+1003.143208}{2} \times 874.0605963$

$\mathcal{A}\ \square ABCD = 1131082.256$

$\boxed{\mathcal{A}\ ABCD = 1131082\ m^2}$

AREA OF PLOT : 5160 *PLOTS*

$\mathcal{A}parcelle = \frac{1131082}{5160}$

$\boxed{\mathcal{A}Parcelle = 219\ m^2}$

Heron formula for $\triangle ADC$

Calculating d_2

$$\frac{\sin\alpha}{1050} = \frac{\sin\beta}{d_2} \Rightarrow d_2 = 1050 \cdot \frac{\sin\beta}{\sin\alpha}$$

$d_2 = 1050 \cdot \frac{\sin 27^\circ.47467203}{\sin 28^\circ.8753967}$

$d_2 = 1003.143208$

The sides : $1050, 1810, 1003.143208$

Calculating s:

$s = \frac{1050+1810+1003.143208}{2}$

$s = 1931.571604$

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Area $\mathcal{A}\triangle ADC = \sqrt{1931.571604 \times 881.571604 \times 121.571604 \times 928.428396}$

Area $\mathcal{A}\triangle ADC = 438403.9751$

Rounding:

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